Chapter 1

extra notes on Lesson 1

1.1 Some Additional Remarks

Remark 1.1.1 *The word "data" is used in a very general sense, these days. It may mean different thing in different context or industry. The computer world the word "data" is used to mean any computer file. In IT industry, it is used to mean information.*

Remark 1.1.2 *A better name for* $N-$*value is the population size.*

Remark 1.1.3 *Currently, a major source of* **sampling bias** *is the cell phone population. Cell phones are not listed and a lot of people do not have aland phone. So, a sample based on land phone listing is biased.*

1.2 Additional problems from Lesson 1

Exercise 1.3 To estimate population of butterflies, 1500 butterflies were tagged in Lawrence. After another two weeks, 1800 butterflies were recaptured. Out of them 300 were tagged. Estimate the population of the butterflies in Lawrence.

Exercise 1.4 To estimate the snake population in a forest area, 78 snakes were captured and tagged. After a few weeks, 112 snakes were caught again and out of them 26 were tagged last time. Estimate the snake population.

Exercise 1.5 To estimate the alligator population in a certain area in Florida, 314 alligator were captured and tagged. After a few months again, 628 alligators were captured and out of them 157 were tagged last time. Estimate the alligators population.

Exercise 1.6 To estimate the bison population in an area in California, 228 bisons were tagged on a day. After a few months again, 285 bisons were captured. Out of them 57 were tagged last time. Estimate the bison population in this area.

Exercise 1.7 To estimate the rhino population in an area in Africa, 129 rhinos were tagged on a day. After a few months again, 215 bisons were captured. Out of them 43 were tagged last time. Estimate the rhino population in this area.

Chapter 2

extra notes on Lesson 2

None at this time.

Chapter 3

extra notes on Lesson 3

None at this time.

Chapter 4

extra notes on Lesson 4

4.1 4.2 The Multiplication Rule of Counting

Ordered Selection

Exercise 4.2 Suppose there are 14 tennis players are competing is a tournament for the top three positions. How many outcomes are possible?

Solution: Since, order matters here, this is an ordered selection of 3 from 14. So, Answer

$$={}_{14}P_3 = \frac{n!}{(n-r)!} = \frac{14!}{(14-3)!} = \frac{14!}{11!} = \frac{1.2.3.4.5\cdots 13.14}{1.2.3\cdots 11} = 12.13.14 = 2184.$$

Exercise 4.3 Mathematics department has funds for awards for the best four teachers this year. Awards have cash values of \$5,000, \$3000, \$2000 and \$1000. There are 37 teachers in the math department. How many selection of four winners is possible?

Solution: Since, order matters here, this is an ordered selection of 4 from 37. So, Answer

$$={}_{37}P_4 = \frac{n!}{(n-r)!} = \frac{37!}{(37-4)!} = \frac{37!}{33!} = \frac{1.2.3\cdots 36.37}{1.2.3\cdots 33} = 34.35.36.37 = 1585080$$

Unordered Selection

Exercise 4.4 Suppose there are 14 applications for three positions in the college office. The positions are all at the same level and pay. How many selection is possible?

Solution: Since all positions are alike, this is an unordered selection of 3 from 14. So,

$$Answer ={}_{14}C_3 = \frac{n!}{(n-r)!r!} = \frac{14!}{(14-3)!3!} = \frac{14!}{11!3!} = \frac{14.13.12}{1.2.3} = 364.$$

Exercise 4.5 A soccer club has 17 players. Eleven has to be selected for a soccar match. How many selection is possible?

Solution: Since, order of selection is irrelevent, this is an unordered selection of 11 from 17.

$$={}_{17}C_{11} = \frac{n!}{(n-r)!r!} = \frac{17!}{(17-11)!11!} = \frac{17!}{6!11!} = \frac{12.13.14.15.16.17}{6!} = 12376$$

Exercise 4.6 A basket ball team has 21 players. Five are selected for a match. How many selection is possible?

Solution: Since, order of selection is irrelevent, this is an unordered selection of 21 from 5.

$$={}_{21}C_5 = \frac{n!}{(n-r)!r!} = \frac{21!}{(21-5)!5!} = \frac{21!}{16!5!} = \frac{17.18.19.20.21}{5!} = 20349.$$

Exercise 4.7 Psychology department has funds for awards, of cash value \$2000 each, for best 5 teachers. There are 44 teachers in the department. How many selection of five winners is possible?

Solution: Since, order of selection is irrelevent, this is an unordered selection of 5 from 44. So, Answer

$$={}_{44}C_5 = \frac{n!}{(n-r)!r!} = \frac{44!}{(44-5)!5!} = \frac{37!}{39!5!} = \frac{40.41.42.43.44}{5!} = 1086008$$

Stand Alone use of Multiplication Rule

Exercise 4.8 Suppose there are three faculty positions in math department in statistics, algebra and geometry. For the statistics position there are 9 appilications, for the algebra position there are 12 appilications, for the geometry position there are 13 appilications. How many different ways is possible to fill these three positions?

Solution: The selection for these three positions have nothing to do with each other. So, permutaion (ordered selection) or combination (unordered selection) formulas do not apply. We use Multiplication Rule as a stand alone tool and make a table:

$position$	$number\ of\ choice$
$statistics$	9
$algebra$	12
$geometry$	13
$answer = product =$	1404

Exercise 4.9 A candidate for a elected position dresses carefully. In his wardrobe, he has 5 pants, 4 shirts, 7 jackets, 8 ties and 4 pairs of shoes. How may different ways he can dress?

Solution: He has to pick one of each item. These items are selected from seperate 'pots'. So, permutaion (ordered selection) or combination (unordered selection) formulas do not apply. We use Multiplication

Rule as a stand alone tool and make a table:

$Item$	$number\ of\ choice$
$pants$	5
$shirts$	4
$jackets$	7
$ties$	8
$answer = product =$	1120

Exercise 4.10 A football team has 43 offense player and 38 defense players. Eleven from each needs to be selected for a game. How many selection is possible for such a game.

Solution: Offense and defense playesr are selected from seperate 'pots'. We use Multiplication Rule as a stand alone tool and make a table:

$select\ 11\ offense\ from\ 43$	${}_{43}C_{11} = 5752004349$
$select\ 11\ defense\ from\ 38$	${}_{38}C_{11} = 1203322288$
$answer = product =$	$too\ big$

4.11 4.3 : Probability

Exercise 4.12 In a certain county, following is the distribution of population:

ethnicity	*W*	*H*	*AA*	*A*	*O*
percent	62	14	13	5	6

Here W=White, H=Hispanic, AA= African American, A= Asian, O=Others. A jury is selcted at random. So, the sample space is

$$S = \{W, H, AA, A, O\}$$

and the probabilty distribution is given by the above table.

1. What is the probability that the jury will a hispanic or asian? $Answer = P(H, A) = .14 + .05$.

2. What is the probability that the jury will not be an asian? $Answer = P(notW) = P(H, AA, A, O) = .14 + .13 + .05 + .06$.

Exercise 4.13 An arbitrary spot is selected in a swamp. The depth (in feet) of water in the swamp is has the following probability distribution:

depth	0+	1+	2+	3+	4+	5+	6+	7+	8+
probability	.1	.2	.09	.17	.13	.11	.08	.07	.05

So, here the sample space is

$$S = \{0+, 1+, 2+, 3+, 4+, 5+, 6+, 7+, 8+\}.$$

1. What is the probability that the depth at an arbitrary spot is less than three feet?

   $Answer = P(Less\ than\ 3) = P(0+, 1+, 2+) = .1 + .2 + .09 = .39$

2. What is the probability that the depth at an arbitrary spot is 3 feet or higher?

   $Answer = P(3\ feet\ or\ higher) = P(3+, 4+, \cdots, 8+) = .17 + .13 + .11 + .08 + .07 + .05 = .61$

Exercise 4.14 A Van pool can carry 7 people. Following is the distribution of number of riders in the van on a given day.

number of	1	2	3	4	5	6	7
probability	0	.12	.22	.23	.28	.08	.07

So, here the sample space is

$$S = \{1, 2, 3, 4, 5, 6, 7\}.$$

1. What is the probability that there will be at most 4 riders?

   $Answer = P(at\ most\ 4) = P(1, 2, 3, 4) = 0 + .12 + .22 + .23 = .57.$

2. What is the probability that there will be less than 4 riders?

   $Answer = P(less\ than\ 4) = P(1, 2, 3) = 0 + .12 + .22 = .34.$

3. What is the probability that there will be more than 4 riders?

   $Answer = P(more\ than\ 4) = P(5, 6, 7, 8) = .23 + .28 + .08 + .07 = .66.$

4. What is the probability that the vam will not be full on a particular day?

   $Answer = P(not\ full) = P(0, 1, 2, 3, 4, 5, 6) = 0 + .12 + .22 + .23 + .28 + .08 = .93$

Exercise 4.15 Following is the distribution of hourly wages (in whole dollars) earned by workers in an industry:

wage	7	8	9	10	11	12	13	14	15	16	17	18	19	20
probability	.04	.06	.07	.09	.11	.12	.14	.11	.09	.08	.04	.03	.01	.01

An employee is selected at random. So, here the sample space is

$$S = \{7, 8, 9, \ldots, 19, 20\}.$$

1. What is the probability that randomly selected worker makes less than 10 dollars an hour?

   $Answer = P(less\ than\ 10) = P(7, 8, 9) = .04 + .06 + .07 = .18$

2. What is the probability that randomly selected worker makes at least \$10 an hour?

   $Answer = P(at\ least\ 10) = P(10, 11, 12, \ldots, 19, 20) = .09 + .11 + .12 + \cdots + .01 + .01 = .82$

3. What is the probability that randomly selected worker makes between \$12-\$16 an hour?

   $Answer = P(12\ to\ 16) = P(12, 13, 14, 15, 16) = .12 + .14 + .11 + .09 + .08 = .54$

Exercise 4.16 In a school district, the distribution of number of students in a class has the following probability distribution:

number	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
prob	.03	.04	.06	.07	.10	.12	.13	.11	.09	.07	.06	.04	.03	.02	.02	.01

A child is selected at random from the school district. So, here the sample space is

$$S = \{8, 9, 10, \ldots, 19, 20, 21, 22, 23\}.$$

1. What is the probability that a child will be in a class of at least 20?

   $Answer = P(at\ least\ 20) = .03 + .02 + .02 + .01 = .08$

2. What is the probability that a child will be in a class of at most 10?

   $Answer = P(at\ most\ 10) = .03 + .04 + .06 = .13$

3. What is the probability that a child will be in a class of less than 10?

   $Answer = P(less\ than\ 10) = P(8, 9) = .03 + .04 = .07$

4.17 4.5 The Complement of an Event

Exercise 4.18 In a county, 38 percent of the community is a minority. What is the probability that a randomly selected jury will not be a minority?

Solution: Let E be the event that the jury will be a minority. Then $P(E) = .38$ Therefore, the answer is

$$P(not\ E) = 1 - P(E) = 1 - .38 = .62$$

Exercise 4.19 In a school district, probability that a student will be in a class of less than 10 students is .27. What is the probability that a randomly selected student will be a class 10 or more?

Solution: Let E be the event that the student will be in a class of less that 10 students. Then, $P(E) = .27$. So, the answer is

$$P(not\ E) = 1 - P(E) = 1 - .27 = .73.$$

Exercise 4.20 In a swamp, probablity that the depth at a random spot is higher than 4 feet is .17. What is the probability that at a random spot, the depth is four feet or less?

Solution: Let E be the event that at a random spot the depth is higher than 4 feet. So, $P(E) = .17$. So, the answer is

$$P(not\ E) = 1 - P(E) = 1 - .17 = .83.$$

Exercise 4.21 It is known that 43 percent of the work force in a town earns more than \$37,000 annualy. What is the probability that a randomly selected working person would make at most \$37,000 annualy?

Solution: Let E be the event that a randomly selected working person would make more than \$37,000 annualy. Therefore, $P(E) = .43$. Therefore, the answer is

$$P(not\ E) = 1 - P(E) = 1 - .43 = .57.$$

Exercise 4.22 It is known that you can get an empty seat in the bus 64 percent of the rides. What is the probability that on a particular ride would not get a seat?

Solution: Let E be the event that you get an empty seat in the bus. So, $P(E) = .64$. Therefore, the answer is

$$P(not\ E) = 1 - P(E) = 1 - .64 = .36$$

4.23 4.6 Independent Events

Exercise 4.24 Suppose you went for a job interview in Lawrence and another one in Kansas City. Probability of that you will get the job in Lawrence is .25 and the probability of that you will get the job in Kansas City is .33. It is reasonable to assume independence.

1. What is the probability that you will get both the jobs?

   Solution: Let E be the event that get the job in Lawrence and F be the event that get the job in Kansas City. So,

   $$P(E) = .25 \qquad and \qquad P(F) = .33$$

   Answer: $P(Both) = P(E\ and\ F) = P(E)P(F) = .25 * .33 = .0825$

2. What is the probability that you will get neither?

   Solution: We have

   $$P(not\ E) = 1-P(E) = 1-.25 = 75, \qquad and \qquad P(not\ F) = 1-.33 = .67$$

   Answer: $P(Neither) = P((not\ E)\ and\ (not\ F)) = P(not\ E)P(not\ F) = .75 * .67 = .5025$

Exercise 4.25 You are taking the Topic course in KU and your brother is taking the same course in MU. The probability that you will get an A is .18 and the probability that your brother will get an A is .21.

1. What is the probability that both of you will get and A.

   Solution: Let E be the event that you will get an A and F be the event that your brother will get an A. Then,

   $$P(E) = .18 \qquad and \qquad P(F) = .21.$$

Answer: $P(Both) = P(EandF) = P(E) * P(F) = .18 * .21 = .0378$.

2. What is the probability that none of you will get an A. We have

$$P(not\ E) = 1 - .18 = .82 \qquad and \qquad P(not\ F) = 1 - .21 = .79.$$

Answer: $P(Neither) = P((not\ E)and(not\ F)) = P(not\ E) * P(not\ F)) = .82 * 79 = .6478$

Exercise 4.26 Probability that you will receive a call from a sibling this week is .35 and the that you will receive a call from a parent this week is .43. What is the probability that you receive a call from both, this week. (Assume independence.)

Solution: Let E be the event that you will receive a call a sibling this week and F be the event that you will receive a call a parent. Then

$$P(E) = .35 \qquad and \qquad P(F) = .43.$$

Answer: $P(Both) = P(E\ and\ F) = P(E) * P(F) = .35 * .43 = .1505$.

Exercise 4.27 Probability that it will rain in Lawrence today is .22 and probability that it will rain today at your home town is .40. What is the probability that it will rain in both places?

Solution: Let E be the event that it will rain in Lawrence today and F be the event that it will rain at your home town today.

Then

$$P(E) = .22 \qquad and \qquad P(F) = .40.$$

Answer: $P(Both) = P(E\ and\ F) = P(E) * P(F) = .22 * .40$

Exercise 4.28 According to the poll, probability that a person would vote for Candidate-A is .43.

1. What is the probability that both you and I would vote for Candidate-A? (We can assume independence because you and I do not influence each other.)

   Solution: Let E be the event that you will vote for Candidate-A and let F be the event that I will vote for Candidate-A. Then $P(E) = P(F) = .43$.

   Answer: $P(Both) = P(E\ and\ F) = P(E) * P(F) = .43 * .43 = .1849$

2. What is the probability that neither you nor I would vote for Candidate-A?

   Solution: We have

   $$P(not\ E) = 1 - .43 = .57 \qquad and \qquad P(not\ F) = 1 - .43 = .57$$

   Answer: $P(Neither) = P((not\ E)\ and\ (not\ F)) = P(not\ E) * P(not\ F) = .57 * .57 = .3249$

Chapter 5

extra notes on Lesson 5: Normal Distribution

5.1 5.2. Normal Distribution

5.1.1 Sigma Distance

Exercise 5.2 Suppose test scores in math 105 has a distribution with mean $\mu = 73$ and standard deviation $\sigma = 21$ points. You got 89 points and your friend got 71 points. How many standard deviation apart are your scores?

Answer: $= \frac{89-71}{\sigma} = \frac{89-71}{21} = 18/21 = .8571$

Exercise 5.3 The distribution of the birth weight of babies has a mean $\mu = 118$ ounce and standard deviation $\sigma = 18$ ounce. The weight of two twin babies are 123 ounce and 111 ounce. How many standard deviaiton apart are these two babies in weight?

Answer: $= \frac{123-111}{\sigma} = \frac{123-111}{18} = 12/18 = .6667$

Exercise 5.4 The distribution of the length (height) of babies at birth has a mean $\mu = 18$ inches and standard deviation $\sigma = 2.5$ inches. The length of two twin babies are 20 inches and 21 inches. How many standard deviaiton apart are these two babies in length?

Answer: $= \frac{21-20}{\sigma} = \frac{21-20}{2.5} = 1/2.5 = .4$

Exercise 5.5 The distribution of housing price in a neighborhood has a mean $\mu = \$138,000$ and standard deviation $\sigma = \$25,000$. You saw two houses and the prices are \$158,000 and \$120,000. How many standard deviaiton apart are these two houses are.

Answer: $= \frac{158{,}000-120{,}000}{\sigma} = \frac{158{,}000-120{,}000}{25{,}000} = 38000/25000 = 1.52$

Exercise 5.6 The distribution of annual household income in a school district has a mean $\mu = \$210,000$ and standard deviation $\sigma = \$68,000$. Two families in a class have annual household income \$255,000 and \$355,000. How many standard deviaiton apart are these two families in household income?

Answer: $= \frac{355{,}000-255{,}000}{\sigma} = \frac{100{,}000}{68{,}000} = 100000/68000 = 1.4706$

5.6.1 Probability Problems

Exercise 5.7 Waiting time at the check out counter in a grocery store has normal distribution with mean $\mu = 180$ seconds and standard deviation $\sigma = 70$ seconds. What is the probability that next time you will have to wait at least 200 seconds.

Solution: Supose X is the waiting time. We need to compute $P(200 \leq X)$. We have

$$P(200 \leq X) = P\left(\frac{200-\mu}{\sigma} \leq \frac{X-\mu}{\sigma}\right) = P\left(\frac{200-180}{70} \leq Z\right)$$

$$= P(.29 \leq Z) = 1 - P(Z \leq .29) = 1 - .6141 = .3859.$$

Exercise 5.8 Miles X that a full tank of gas in my car gives has normal distribution with mean $\mu = 315$ and standard deviation $\sigma = 11$ miles. What is the probability that next time I fill the tank, I would be able to drive between 310 to 330 miles?

Solution: We need to compute $P(310 \leq X \leq 330)$. We have

$$P(310 \leq X \leq 330) = P\left(\frac{310-\mu}{\sigma} \leq \frac{X-\mu}{\sigma} \leq \frac{330-\mu}{\sigma}\right)$$

$$= P\left(\frac{310-315}{11} \leq Z \leq \frac{330-315}{11}\right) = P(-.45 \leq Z \leq 1.36)$$

$$= P(Z \leq 1.36) - P(Z \leq -.45) = 0.9131 - 0.3264 = 0.5867.$$

Exercise 5.9 The amount X of water used when a person takes a shower has a normal distribution with mean $\mu = 30$ gallons and standard deviation $\sigma = 16$ gallons. What is the probability that next time you take a shower, you will use less than 20 gallons of water?

Solution: We need to compute $P(X \leq 20)$. We have,

$$P(X \leq 20) = P\left(\frac{X-\mu}{\sigma} \leq \frac{20-\mu}{\sigma}\right) = P\left(Z \leq \frac{20-30}{16}\right)$$
$$= P\left(Z \leq -.63\right) = 0.2643.$$

Exercise 5.10 The time X that a child spends watching TV on weekends has a normal distribution with mean $\mu = 340$ minutes and standard deviation $\sigma = 95$ minutes. What is the probability that your child/sibling would watch TV at least 400 minutes this weekend?

Solution: We need to compute $P(400 \leq X)$. We have,

$$P(400 \leq X) = P\left(\frac{400-\mu}{\sigma} \leq \frac{X-\mu}{\sigma}\right) = P\left(\frac{400-340}{95} \leq Z\right)$$
$$= P\left(.63 \leq Z\right) = 1 - P\left(Z \leq .63\right) = 1 - .7357 = .2643.$$

Exercise 5.11 The amount of time X a person spends to read a news article on internet has a mean $\mu = 3$ minutes and standard deviation $\sigma = 1.5$ minutes. What is the probability that the next news article I read will take at most 2 minutes?

Solution: We need to compute $P(X \leq 2)$. We have,

$$P(X \leq 2) = P\left(\frac{X-\mu}{\sigma} \leq \frac{2-\mu}{\sigma}\right)$$
$$= P\left(Z < \frac{2-3}{1.5}\right) = P\left(Z < -.67\right) = 0.2514.$$

Chapter 6

Binomial Random Variables

Exercise 6.1 It is believed proportion of voters (in a county) who vote by absentee ballot is $p = .18$. You sample 3025 voters. What is the probability that number of voters who would vote by absentee ballots will be at least 570?

Solution: Let X be the number of voters who would vote by absentee ballots. We compute $P(570 \leq X)$. We have, $n = 3025$. So,

$$mean = np = 3025 * .18 = 544.5$$

and standard deviation

$$\sigma = \sqrt{n * p * (1-p)} = \sqrt{3025 * .18 * (1 - .18)} = 21.1303.$$

So,

$$P(570 \leq X) = P\left(\frac{570-\mu}{\sigma} \leq \frac{X-\mu}{\sigma}\right) = P\left(\frac{570 - 544.5}{21.1303} \leq Z\right)$$

$$= P\left(1.21 \leq Z\right) = 1- = P\left(Z < 1.21\right) = 1 - .8869 = .1131.$$

Exercise 6.2 About 27 percent of the population take flu shots. In a class of 750 students, what is the probability that atmost 200 would have taken a flu shot.

Solution: Let X be the number of success. We compute $P(X \leq 200)$. We have, $p = .27$ and $n = 750$. So,

$$mean = np = 750 * .27 = 202.5$$

and standard deviation

$$\sigma = \sqrt{n * p * (1-p)} = \sqrt{750 * .27 * (1 - .27)} = 12.1583.$$

So,

$$P(X \leq 200) = P\left(\frac{X-\mu}{\sigma} \leq \frac{200-\mu}{\sigma}\right) = P\left(Z \leq \frac{200-202.5}{12.1583}\right)$$

$$= P(Z \leq -.21) = 0.4168.$$

Exercise 6.3 It is believed that 35 percent of the population in a county shop in health food market. If you sample 1100 individuals, what is the probability that at least 400 would shop in health food market?

Solution: Let X be the number of success. We compute $P(400 \leq X)$. We have, $p = .35$ and $n = 1100$. So,

$$mean = np = 1100 * .35 = 385$$

and standard deviation

$$\sigma = \sqrt{n * p * (1-p)} = \sqrt{1100 * .35 * (1 - .35)} = 15.8193.$$

So,

$$P(400 \le X) = P\left(\frac{400-\mu}{\sigma} \le \frac{X-\mu}{\sigma}\right) = P\left(\frac{400-385}{15.8193} \le Z\right)$$

$$= P\,(0.95 \le Z) = 1 - P\,(Z < 0.95) = 1 - 0.8289 = 0.1711.$$

Exercise 6.4 It is known that 78 percent of the microwave ovens lasts more that five years. A SQC inspector samples 1500 microwaves. What is the probability that the number of microwaves in this sample that lasted more than five years will be between 1150 and 1200?

Solution: Let X be the number of success. We compute $P(1150 \le X \le 1200)$. We have, $p = .78$ and $n = 1500$. So,

$$mean = np = 1500 * .78 = 1170$$

and standard deviation

$$\sigma = \sqrt{n * p * (1-p)} = \sqrt{1500 * .78 * (1 - .78)} = 16.0437.$$

So,

$$P(1150 \le X \le 1200) = P\left(\frac{1150-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{1200-\mu}{\sigma}\right)$$

$$= P\left(\frac{1150-1170}{16.0437} \le Z \le \frac{1200-1170}{16.0437}\right) = P\,(-1.25 \le Z \le 1.87)$$

$$= P\,(Z \le 1.87) - P\,(Z \le -1.25) = 0.9693 - 0.1056 = 0.8637.$$

Exercise 6.5 It is known that a vaccine may cause fever as side effect, after one takes the shot. The producer of the vaccine claims that only 11 percent of those who take the shot experience such side effects. You sample 978 individuals. What is the probability that at least 130 will experience such side effects.

Solution: Let X be the number of success. We compute $P(130 \leq X)$. We have, $p = .11$ and $n = 978$. So,

$$mean = np = 978 * .11 = 107.58$$

and standard deviation

$$\sigma = \sqrt{n * p * (1-p)} = \sqrt{978 * .11 * (1 - .11)} = 9.7850.$$

So,

$$P(130 \leq X) = P\left(\frac{130-\mu}{\sigma} \leq \frac{X-\mu}{\sigma}\right) = P\left(\frac{130-107.58}{9.7850} \leq Z\right)$$

$$= P\left(2.29 \leq Z\right) = 1 - P\left(Z \leq 2.29\right) = 1 - 0.9890 = 0.011.$$

Chapter 7

extra problems on Lesson 7

Exercise 7.1 Suppose the teacher says mean of the test-1 grade in thsi class is $\mu = 77$ and the standard deviation $\sigma = 24.5$ points. You take a sample of 81 students and find the probability that the sample mean $\bar{X}$ would exceed 85?

Solution: We compute $P(85 < \bar{X})$. First, the mean of $\bar{X} = \mu = 77$. The standard deviation of $\bar{X}$ is

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{24.5}{\sqrt{81}} = 2.7222.$$

So,

$$P(85 < \bar{X}) = P\left(\frac{85 - \mu}{\sigma_{\bar{X}}} \leq \frac{\bar{X} - \mu}{\sigma_{\bar{X}}}\right) = P\left(\frac{85 - 77}{2.7222} \leq Z\right)$$

$$= P\left(2.94 \leq Z\right) = 1 - P\left(Z \leq 2.94\right) = 1 - 0.9984 = 0.0016.$$

Exercise 7.2 The mean salary X of the university professors in a state is $\mu = \$65,000$ and standard deviation $\sigma = \$14,000$. You collect a sample of 75 professors. What is the probability that sample mean salary of these 75 professors would be above \$60,000.

Solution: We compute $P(60,000 < \bar{X})$. First, the mean of $\bar{X} = \mu = 65,000$. The standard deviation of $\bar{X}$ is

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{14,000}{\sqrt{75}} = 1616.5808.$$

So,

$$P(60,000 < \bar{X}) = P\left(\frac{60,000 - \mu}{\sigma_{\bar{X}}} \leq \frac{\bar{X} - \mu}{\sigma_{\bar{X}}}\right) = P\left(\frac{60000 - 65000}{1616.5808} \leq Z\right)$$

$$= P\left(-3.09 \leq Z\right) = 1 - P\left(Z \leq -3.09\right) = 1 - 0.001 = 0.999.$$

Exercise 7.3 The time X that a child spends watching TV on weekends has a normal distribution with mean $\mu = 330$ minutesand standard deviation $\sigma = 95$ minutes. You sample 50 kids in a school. What is the probability that the sample time $\bar{X}$ that these kids watch TV on a weekend will be less than 300 minutes.

Solution: We compute $P(\bar{X} < 300)$. The mean of $\bar{X} = \mu = 330$. The standard deviation of $\bar{X}$ is

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{95}{\sqrt{50}} = 13.4350.$$

So,

$$P(\bar{X} < 300) = P\left(\frac{\bar{X} - \mu}{\sigma_{\bar{X}}} < \frac{300 - \mu}{\sigma_{\bar{X}}}\right) = P\left(Z < \frac{300 - 330}{13.4350}\right)$$

$$= P\left(Z < -2.23\right) = 0.0129.$$

Exercise 7.4 The weight X of fish in a lake has mean $\mu = 12$ pounds and standard deviation $\sigma = 4.5$ pounds. Suppose you catch 150 fish. What is the probability that **total weight** of fish will be less than 1900 pounds?

Solution: Like this one, some problems are posed in terms of total, instead of sample mean $\bar{X}$. First note sample mean

$$\bar{X} = \frac{total}{n}.$$

We have $n = 150$, we have to compute

$$P(total < 1900) = P\left(\frac{total}{n} < \frac{1900}{n}\right) = P\left(\frac{total}{n} < \frac{1900}{n}\right)$$

$$= P\left(\bar{X} < \frac{1900}{150}\right) = P\left(\bar{X} < 12.6667\right).$$

Also, the mean of $\bar{X} = \mu = 12$. The standard deviation of $\bar{X}$ is

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{4.5}{\sqrt{150}} = 0.3674.$$

So,

$$P(\bar{X} < 12.6667) = P\left(\frac{\bar{X} - \mu}{\sigma_{\bar{X}}} \leq \frac{12.6667 - \mu}{\sigma_{\bar{X}}}\right) = P\left(Z < \frac{12.6667 - 12}{0.3674}\right)$$

$$= P\left(Z \leq 1.81\right) = 0.9649.$$

Exercise 7.5 The amount X of water used when a person takes a shower has a normal distribution with mean $\mu = 30$ gallons and standard deviation $\sigma = 16$ gallons. Suppose 36 people take a shower. What is the probability that total of more than 900 gallons of water will be used by these 36 people.

Solution: The problem is posed in terms of total, instead of sample mean $\bar{X}$. First note sample mean

$$\bar{X} = \frac{total}{n}.$$

We have $n = 36$, we have to compute

$$P(900 < total) = P\left(\frac{900}{n} < \frac{total}{n}\right) = P\left(\frac{900}{n}\frac{total}{n} <\right)$$

$$= P\left(\frac{900}{36} < \bar{X}\right) = P\left(25 < \bar{X}\right).$$

Also, the mean of $\bar{X} = \mu = 30$. The standard deviation of $\bar{X}$ is

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{16}{\sqrt{36}} = 2.6667.$$

So,

$$P(25 < \bar{X}) = P\left(\frac{25-\mu}{\sigma_{\bar{X}}} < \frac{\bar{X}-\mu}{\sigma_{\bar{X}}}\right) = P\left(\frac{25-30}{2.6667} < Z\right)$$

$$= P\left(-1.87 < Z\right) = 1 - P\left(Z < -1.87\right) = 1 - 0.0307 = 0.9693.$$

Chapter 8

Estimation

8.1 Interval Estimation: zInterval

Exercise 8.2 The amount X of water used when a person takes a shower has a normal distribution with mean μ gallons and known standard deviation $\sigma = 16$ gallons. To estimate mean μ you take a sample of size 82 and found sample mean $\bar{x} = 33$ gallons. Compute a 99 percent confidence interval.

Solution: We use TI-83/84. We get

$$LEP = 28.449, \quad REP = 37.551,$$

and

$$MOE = \frac{REP - LEP}{2} = \frac{37.551 - 28.449}{2} = 4.551.$$

8.3 t-Interval μ: whenσ is unknown

None at this time.

8.4 1-propZInterval for p

Exercise 8.5 On the last poll before 2008 election, Gallup polled 2824 registered voters. Out of them, 1497 favored Barack Obama and 1130 favored John McCain.

1. First compute the conservative MOE for this poll at 95 percent level of confidence.

   Solution: Here level of confidence $1 - \alpha = .95$. So, $\alpha/2 = .025$ and from the table $z_{\alpha/2} = 1.96$. Therefore, Conservative margin of error:
   $$E = \frac{z_{\alpha/2}}{\sqrt{4n}} = \frac{1.96}{\sqrt{4 * 2824}} = 0.0184$$
   or 1.84 percent.

2. Compute the proportion of success for Obama and McCain.

   Solution: For Obama, proportion of success
   $$\bar{X} = \frac{X}{n} = \frac{1497}{2824} = .5301$$
   and for McCain
   $$\bar{Y} = \frac{Y}{n} = \frac{1130}{2824} = .4001.$$

3. Use TI-83/84 to compute a 95 pecent confidence interval for Barck Obama.

   Solution: For Obama, from TI-84, we have
   $$LEP = .51169, \qquad REP = .54851.$$

4. Use TI-83/84 to compute a 95 pecent confidence intervalfor John McCain.

Solution: For McCain, from TI-84, we have

$$LEP = .38207, \qquad REP = .41821.$$

Chapter 9

Testing of Hypotheses

None at this time.

Chapter 10

Fibonacci

Exercise 10.1 Use the formula

$$F_N = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^N - \left(\frac{1-\sqrt{5}}{2}\right)^N}{\sqrt{5}}$$

to compute $F_{20}, F_{25}, F_{30}, F_{100}$.

Solution:

$$F_{20} = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^{20} - \left(\frac{1-\sqrt{5}}{2}\right)^{20}}{\sqrt{5}} = 6765$$

Now

$$F_{25} = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^{25} - \left(\frac{1-\sqrt{5}}{2}\right)^{25}}{\sqrt{5}} = 75,025$$

Then

$$F_{30} = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^{30} - \left(\frac{1-\sqrt{5}}{2}\right)^{30}}{\sqrt{5}} = 832,040$$

Finally,

$$F_{100} = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^{100} - \left(\frac{1-\sqrt{5}}{2}\right)^{100}}{\sqrt{5}} = \textit{too big}$$

10.1.1 Proof by Method of Mathematical Induction

In mathematics, we do not accept validity of any any statemnt without a proof. There is a well accepted mothod of proof the is called **Method of Mathematical Induction**, as follows:

Proof by Method of Mathematical Induction: Suppose $P(n)$ is a statement or a propostion that depends on integers $n = 1, 2, 3, \ldots$. Suppose

1. Suppose $P(1)$ is valid.
2. Suppose, for all integers $r = 1, 2, \ldots$, the valdity of $P(1), P(2), \ldots, P(r)$ implies the validity of $P(r + 1)$.

Then $P(n)$ is valid for all $n = 1, 2, \ldots$.

We use this method to prove the formula for Fibonacci sequence

$$F_N = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^N - \left(\frac{1-\sqrt{5}}{2}\right)^N}{\sqrt{5}}$$

Proof. Recall, that the Fibonacci sequence F_n is defined by

$$F_1 = 1, F_2 = 1 \quad and \quad for \quad n \geq 3 \quad F_n = F_{n-2} + F_{n-1}.$$

Let $P(n)$ be the statement that the formula is valid for the integer n. Clearly,

$$F_1 = 1 = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^1 - \left(\frac{1-\sqrt{5}}{2}\right)^1}{\sqrt{5}}$$

So, the formula is valid for $r = 1$. Also,

$$F_2 = 1 = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^2 - \left(\frac{1-\sqrt{5}}{2}\right)^2}{\sqrt{5}}$$

So, the formula is valid for $r = 2$. Now assume that the formula is valid for $n = 1, 2, \ldots, r-1, r$. In particular,

$$F_{r-1} = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^{r-1} - \left(\frac{1-\sqrt{5}}{2}\right)^{r-1}}{\sqrt{5}} \qquad \textit{and} \qquad F_r = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^{r} - \left(\frac{1-\sqrt{5}}{2}\right)^{r}}{\sqrt{5}}$$

So, $F_{r+1} = F_{r-1} + F_r =$

$$= \frac{\left(\frac{1+\sqrt{5}}{2}\right)^{r-1} - \left(\frac{1-\sqrt{5}}{2}\right)^{r-1}}{\sqrt{5}} + \frac{\left(\frac{1+\sqrt{5}}{2}\right)^{r} - \left(\frac{1-\sqrt{5}}{2}\right)^{r}}{\sqrt{5}}$$

$$= \frac{\left[\left(\frac{1+\sqrt{5}}{2}\right)^{r-1} + \left(\frac{1+\sqrt{5}}{2}\right)^{r}\right] - \left[\left(\frac{1-\sqrt{5}}{2}\right)^{r-1} + \left(\frac{1-\sqrt{5}}{2}\right)^{r}\right]}{\sqrt{5}}$$

$$= \frac{\left(\frac{1+\sqrt{5}}{2}\right)^{r-1}\left[1 + \left(\frac{1+\sqrt{5}}{2}\right)\right] - \left(\frac{1-\sqrt{5}}{2}\right)^{r-1}\left[1 + \left(\frac{1-\sqrt{5}}{2}\right)\right]}{\sqrt{5}}$$

$$= \frac{\left(\frac{1+\sqrt{5}}{2}\right)^{r-1}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{2}\right] - \left(\frac{1-\sqrt{5}}{2}\right)^{r-1}\left[\left(\frac{1-\sqrt{5}}{2}\right)^{2}\right]}{\sqrt{5}}$$

$$= \frac{\left(\frac{1+\sqrt{5}}{2}\right)^{r+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{r+1}}{\sqrt{5}}$$

So, the statement is valid for $n = r+1$. So, by method of mathematical induction the formula is valis for all integers $r \geq 1$.

Chapter 11

Population Growth and Sequences

11.1 Linear Growth and Arithmetic Sequence

1. For the Liner growth model, with common difference d, we have two formulas for the size P_n of the $n^{th}-$generation:

$$P_n = P_{n-1} + d \qquad and \qquad P_n = P_1 + (n-1)d.$$

2. Similarly, for an arithmetic sequence $a_1, a_2, a_3, \ldots, a_{n-1}, a_n, \ldots,$ common difference

$$d = a_2 - a_1 = a_3 - a_2 = \cdots = a_n - a_{n-1}$$

and

$$a_n = a_{n-1} + d \qquad and \qquad a_n = a_1 + (n-1)d.$$

Further, the sum

$$a_1+a_2+\cdots+a_n = \frac{n(a_1+a_n)}{2} = \frac{[number\ of\ terms](First + Last)}{2}.$$

Problems on Linear Growth

Exercise 11.2 A population grows according to linear growth model $P_n = P_{n-1} + 17$. The initial population size is $P_1 = 119$.

1. What is the common difference?

   Solution: Compare $P_n = P_{n-1} + 17$ with the formula $P_n = P_{n-1} + d$. So, the common difference $d = 17$.

2. Compute P_8.

   Solution: $P_8 = P_1 + (n-1)d = 119 + (8-1)17 = 238$.

3. Compute P_{11}.

   Solution: $P_{11} = P_1 + (n-1)d = 119 + (11-1)17 = 289$.

4. Compute P_{19}.

   Solution: $P_{19} = P_1 + (n-1)d = 119 + (19-1)17 = 425$.

Exercise 11.3 A population grows according to linear growth model. It is known that the 8^{th} generation population is $P_8 = 140$ and the 15^{th} generation population is $P_{15} = 273$.

1. What is the common difference?

   Solution: We the formula that $P_n = P_1 + (n-1)d$ and we hve

   $$P_8 = 140 = P_1 + 7d \qquad and \qquad P_{15} = 273 = P_1 + 14d.$$

   Subtract the first equation from the later:

   $$273 - 140 = 7d \qquad OR \qquad 133 = 7d \qquad OR \qquad 19 = d.$$

   So, the common difference $d = 19$.

2. Compute P_1.

   Solution: $P_8 = P_1 + 7d = 140$. Thereefore $P_1 + 7 * 19 = 140$ and $P_1 = 140 - 7 * 19 = 7$.

3. Compute P_{13}.

   Solution: Now

   $$P_{13} = P_1 + (n-1)d = 7 + (13-1)19 = 235.$$

4. Compute P_{21}.

   Solution: Now

   $$P_{21} = P_1 + (n-1)d = 7 + (21-1)19 = 387.$$

Exercise 11.4 A population grows according to linear growth model. The first few generation populations are given by $7, 20, 33, 46, \ldots$.

1. What is the common difference?

   Solution: The common difference

   $$d = second - first = 20 - 7 = 13.$$

2. Compute P_1.

   Clearly, $P_1 = 7$.

3. Compute P_9.

   Solution:

   $$P_9 = P_1 + (n-1)d = 7 + 8 * 13 = 111.$$

4. Compute P_{31}.

   Solution:

$$P_{31} = P_1 + (n-1)d = 7 + 30 * 13 = 397.$$

Problems on Arithmetic Sequences

Exercise 11.5 Consider the arithmetic sequence $7, 10, 13, 16, \ldots$.

1. What is the common difference?

   Solution: The common difference

$$d = second - first = 10 - 7 = 3.$$

2. Compute the 9^{th} term a_9.

   Solution:

$$a_9 = a_1 + (n-1)d = 7 + 8 * 3 = 31.$$

3. Compute the 19^{th} term a_{19}.

   Solution:

$$a_{19} = a_1 + (n-1)d = 7 + 20 * 3 = 67.$$

4. Compute the 33^{rd} term a_{33}.

   Solution:

$$a_{33} = a_1 + (n-1)d = 7 + 32 * 3 = 103.$$

Exercise 11.6 Consider the arithmetic sequence $3, 9, 15, 21, \ldots$.

1. What is the the first term a_1 and the common difference?

   Solution: We have $a_1 = 3$ and the common difference

   $$d = second - first = 9 - 3 = 6.$$

2. Compute the 10^{th} term a_{10}.

   Solution:

   $$a_{10} = a_1 + (n-1)d = 3 + 9 * 6 = 57.$$

3. Compute the 20^{th} term a_{20}.

   Solution:

   $$a_{20} = a_1 + (n-1)d = 3 + 19 * 6 = 117.$$

4. Compute the 30^{th} term a_{30}.

   Solution:

   $$a_{30} = a_1 + (n-1)d = 3 + 29 * 6 = 177.$$

Exercise 11.7 consider the arithmetic sequence $a_n = 10 + 9(n-1)$.

1. What is the common difference?

   Solution: Compare $a_n = 10 + 9(n-1)$ with the formula $a_n = a_1 + (n-1)d$. So, $a_1 = 10$ and the common difference $d = 9$.

2. Compute the 6^{th} term a_6.

   Solution:

   $$a_6 = a_1 + (n-1)d = 10 + 5 * 9 = 55.$$

3. Compute the 16^{th} term a_{16}.

   Solution:

   $$a_{16} = a_1 + (n-1)d = 10 + 15 * 9 = 145.$$

4. Compute the 33^{rd} term a_{33}.

   Solution:

   $$a_{33} = a_1 + (n-1)d = 10 + 32 * 9 = 298.$$

Problems on sum

Exercise 11.8 Consider the arithmetic sequence $a_n = 10 + 3(n-1)$.

1. What is the common difference?

   Solution: Compare $a_n = 10 + 3(n-1)$ with the formula $a_n = a_1 + (n-1)d$. So, $a_1 = 10$ and the common difference $d = 3$.

2. Compute the sum of first 100 terms $10 + 13 + 16 + \cdots + 307$.

   Solution: The sum
   $$= \frac{[number\ of\ terms](First + Last)}{2} = \frac{100(307+10)}{2} = 15850.$$

3. Compute the sum of first 200 terms $10 + 13 + 16 + \cdots + 607$

   Solution: The sum
   $$= \frac{[number\ of\ terms](First + Last)}{2} = \frac{200(607+10)}{2} = 61700.$$

4. Compute the sum of first 300 terms $10 + 13 + 16 + \cdots + 907$.

   Solution: The sum
   $$= \frac{[number\ of\ terms](First + Last)}{2} = \frac{300(907+10)}{2} = 137550.$$

11.9 Exponential growth and geometric sequence

1. For the exponential growth model, with common ratio r, we have two formulas for the size P_n of the $n^{th}-$generation:

$$P_n = rP_{n-1} \qquad and \qquad P_n = r^{n-1}P_1.$$

2. Similarly, for a geometric sequence $a_1, a_2, a_3, \ldots, a_{n-1}, a_n, \ldots$, common ratio

$$d = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \cdots = \frac{a_n}{a_{n-1}}$$

and

$$a_n = ra_{n-1} \qquad and \qquad a_n = r^{n-1}a_1.$$

Further, the sum

$$a_1 + a_2 + \cdots + a_n = \frac{a_1(r^n - 1)}{r - 1} = \frac{a_1(1 - r^n)}{1 - r}.$$

Exercise 11.10 A population grows according to exponential growth model $P_n = 1.7P_{n-1}$. The initial population size is $P_1 = 13$.

1. What is the common ratio?

   Solution: Compare $P_n = 1.7P_{n-1}$ with the formula $P_n = rP_{n-1}$. So, the common ratio $r = 1.7$.

2. Compute P_7.

   Solution:

$$P_7 = r^{n-1}P_1 = (1.7)^6 * 13 = 313.7884.$$

3. Compute P_{11}.

   Solution:

$$P_{11} = r^{n-1}P_1 = (1.7)^{10} * 13 = 2620.7921.$$

4. Compute P_{18}.

   Solution:

$$P_{18} = r^{n-1}P_1 = (1.7)^{17} * 13 = 107541.234.$$

Exercise 11.11 A population grows according to exponential growth model. It is known that the 5^{th} generation population is $P_5 = 2856100$ and the 15^{th} generation population is $P_{14} = 30287510.66$.

1. What is the common ratio?

   Solution: We have

$$P_{14} = r^{13}P_1 = 30287510.66 \qquad and P_5 = r^4 P_1 = 2856100.$$

   Divide the first equation by the second, we get

$$r^9 = \frac{30287510.66}{2856100}.$$

   Take the $9^{th}-$root, use the calculator to do it, we get

$$r = 1.3.$$

2. Compute P_1.

   Solution: We have

$$P_5 = 2856100 = r^4 P_1 \qquad OR \qquad 2856100 = (1.3)^4 P_1$$

   So,

$$P_1 = \frac{2856100}{(1.3)^4} = 1000,000.$$

3. Compute P_{12}.

   Solution: We have

$$P_{12} = r^{n-1}P_1 = (1.3)^{11} * 1000000 = 17921603.94$$

4. Compute P_{22}.

   Solution: We have

$$P_{22} = r^{n-1}P_1 = (1.3)^{21} * 1000000 = 247064529.1$$

Exercise 11.12 A population grows according to geometric growth model. The first few generation populations are given by $7, 10.5, 15.75, 23.625, \cdots$.

1.

2. What is the common ratio ?

   Solution: The common ratio

$$r = \frac{P_2}{P_1} = \frac{10.5}{7} = 1.5$$

3. Compute P_2.

   Solution:

$$P_2 = r^{n-1}P_1 = (1.5)^1 * 7 = 10.5,$$

4. Compute P_8.

   Solution:

$$P_8 = r^{n-1}P_1 = (1.5)^7 * 7 = 119.6016$$

5. Compute P_{11}.

   Solution:

$$P_{11} = r^{n-1}P_1 = (1.5)^{10} * 7 = 403.6553$$

Problems on Geometric Sequences

Exercise 11.13 Consider the geometric sequence $7, 6.3, 5.67, 5.103, 4.5927, \ldots$.

1. What is the common ratio?

   Solution:
   $$r = \frac{a_2}{a_1} = \frac{6.3}{7} = .9$$

2. Compute the 7^{th} term a_7.

   Solution:
   $$a_7 = r^{n-1}a_1 = (.9)^6 * 7 = 3.720087$$

3. Compute the 19^{th} term a_{19}.

   Solution:
   $$a_{19} = r^{n-1}a_1 = (.9)^{18} * 7 = 1.0507$$

4. Compute the 23^{rd} term a_{23}.

   Solution:
   $$a_{23} = r^{n-1}a_1 = (.9)^{22} * 7 = .6893.$$

Exercise 11.14 Consider the geometric sequence $3, 3.3, 3.63, 3.993, \ldots$.

1. What is the common ratio?

   Solution:
   $$r = \frac{a_2}{a_1} = \frac{3.3}{3} = 1.1$$

2. Compute the 13^{th} term a_{13}.

   Solution:
   $$a_{13} = r^{n-1}a_1 = (1.1)^{12} * 3 = 9.4153.$$

3. Compute the 23^{th} term a_{23}.

   Solution:

$$a_{23} = r^{n-1}a_1 = (1.1)^{22} * 3 = 24.4208.$$

4. Compute the 33^{th} term a_{33}.

   Solution:

$$a_{33} = r^{n-1}a_1 = (1.1)^{32} * 3 = 63.3413.$$

Exercise 11.15 Consider the geometric sequence $a_n = 1.11^{(n-1)}1000$.

1. What is the common ratio?

   Solution: Compare $a_n = 1.11^{(n-1)}1000$ with the formula $a_n = r^{(n-1)}a_1$. So, the common ration $r = 1.11$.

2. Compute the 5^{th} term a_5.

   Solution:

$$a_5 = 1.11^{(n-1)}1000 = 1.11^4 1000 = 1518.0704$$

3. Compute the 15^{th} term a_{15}.

   Solution:

$$a_{15} = 1.11^{(n-1)}1000 = 1.11^{14}1000 = 4310.44098.$$

4. Compute the 33^{th} term a_{33}.

Problems on sum

Exercise 11.16 Consider the geometric sequence $a_n = 3.3^{(n-1)}10$.

1. What is the common ratio and the first term?

   Solution: Compare $a_n = 3.3^{(n-1)}10$ with the formula $a_n = r^{(n-1)}a_1$. So, the common ration $r = 3.3$.

   The first term $a_1 = 10$.

2. Compute the sum of first 10 terms $10+33+108.9+\cdots+(3.3)^9 10$.

   Solution: The sum

   $$= \frac{a_1(r^n - 1)}{r - 1} = \frac{10(3.3^9 - 1)}{3.3 - 1} = 201784.7148$$

3. Compute the sum of first 20 terms $10+33+108.9+\cdots+(3.3)^{19}10$.

   Solution: The sum

   $$= \frac{a_1(r^n - 1)}{r - 1} = \frac{10(3.3^{19} - 1)}{3.3 - 1} = big$$

4. Compute the sum of first 30 terms $10+33+108.9+\cdots+(3.3)^{29}10$.

   Solution: The sum

   $$= \frac{a_1(r^n - 1)}{r - 1} = \frac{10(3.3^{29} - 1)}{3.3 - 1} = big$$

Exercise 11.17 Consider the geometric sequence $1, 0.5, 0.5^2, 0.5^3, 0.5^4, 0.5^5, \ldots$.

1. What is the common ratio and the first term?

   Solution: The common ratio

   $$r = \frac{second}{first} = \frac{.5}{1} = .5.$$

   The first term $a_1 = 1$.

2. Compute the sum of first 10 terms?

   Solution: The sum of first 10 terms

   $$= \frac{a_1(r^n - 1)}{r - 1} = \frac{1(.5^9 - 1)}{.5 - 1} = 1.9961$$

3. Compute the sum of first 100 terms?

   Solution: The sum of first 100 terms

   $$= \frac{a_1(r^n - 1)}{r - 1} = \frac{1(.5^{99} - 1)}{.5 - 1} = 2\ (approx)$$

4. Compute the sum of first 200 terms?

   Solution: The sum of first 200 terms

   $$= \frac{a_1(r^n - 1)}{r - 1} = \frac{1(.5^{199} - 1)}{.5 - 1} = 2\ (approx)$$

5. What can you say about the sum, approximately, of a large number of terms?

   Solution: Such a sums of large number of terms will be approximately be equal to 2.

   In fact, for any geometric sequence, whenever $0 < r < 1$, the SUM of a large number of terms will approximately be equal to

   $$sum = \frac{a_1}{1 - r}.$$