Math 365: Lesson 4

More from Random Variables

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1 Problems on Random Variables

Exercise 1.1 The following table gives the proportion of credit hours that earned grades F, D, C, B *and* A in KU:

grade	A	B	C	D	F
proportion	.15	.35	.30	.15	.05

Let X represent the points earned for grades A, B, C, D *and* F. Write down the probability distribution of X and compute the mean (or the expected value $E(X)$) and the standard deviation.

Solution: We have $X = 0, 1, 2, 3, 4$ respectively, when the grades are F, D, C, B, A. Therefore, the distribution of X is given by

x	0	1	2	3	4
$p(x) = P(X = x)$	.05	.15	.30	.35	.15

Now, the mean μ is given by

$$\mu = \sum x_i p(x_i) = 0 * .05 + 1 * .15 + 2 * .30 + 3 * .35 + 4 * .15 = 2.4$$

The variance $\sigma^2 =$

$$\sum x_i^2 p(x_i) - mu^2 = 0^2 * .05 + 1^2 * .15 + 2^2 * .30 + 3^2 * .35 + 4^2 * .15 - (2.4)^2 = 1.14$$

The square root of the variance is the standard deviation. So, the standard deviation

$$\sigma = \sqrt{1.14} = 1.0677.$$

Exercise 1.2 Maria's daily income X (in dollars) has the following distribution.

$X = x$	0	100	120	130	140	150
$p(x)$	.14	0.27	0.27	0.18	0.09	0.05

What is Maria's expected daily income and the standard deviation?

Exercise 1.3 The number X of typos in a website has the following probability distribution.

$X = x$	0	1	2	3	4	5
$p(x)$	0.24	0.31	0.23	0.14	0.07	0.01

What is the expected number of typos in a website?

Exercise 1.4 A Van pool can carry 7 people. Following is the distribution of number of riders in the van on a given day.

number of	1	2	3	4	5	6	7
probability	0	.12	.22	.23	.28	.08	.07

Let X be the number of passenger on a day. Find the expected value $E(X)$ (or mean) and the standard deviation of X.

Exercise 1.5 Let X represent the hourly wages (in whole dollars) earned by workers in an industry. Following is the distribution of X,

x $(wages)$	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$p(x)$	.04	.06	.07	.09	.11	.12	.14	.11	.09	.08	.04	.03	.01	.01

Find the expected value $E(X)$ (or mean) and the standard deviation of X.

Exercise 1.6 In a school district, let X represents the number of students in a class. The following is the distribution of X.

number	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
prob	.03	.04	.06	.07	.10	.12	.13	.11	.09	.07	.06	.04	.03	.02	.02	.01

1. What is the probability that X is at least 20?

   $Answer = P(at\ least\ 20) = .03 + .02 + .02 + .01 = .08$

2. Find the expected value $E(X)$ (or mean) and the standard deviation of X.

3. Find the variance σ^2 of X.

4. Find the standard deviation σ of X.